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Level 2 Certificate in Further Mathematics

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Paper 1 8360/1

Mark Scheme

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M Dep	A method mark dependent on a previous method mark being awarded.
B Dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

Paper 1 - Non-Calculator

Q	Answer	Mark	Comments
1(a)	9	B1	
1(b)	$f(x) \geq 7$	B1	Allow $y \geq 7$
2	$\begin{pmatrix} 10 \\ 17 \end{pmatrix}$	B2	B1 For each component $\begin{pmatrix} 10 + 0 \\ 5 + 12 \end{pmatrix}$ scores B1
3	$3x < -9$ or $x < -3$	M1	oe
	-4	A1	SC1 For $x \leq -4$
4(a)	$2(x^2 - x - 20)$	M1	Common factor might be removed later
	$(2x + a)(x + b)$ or $2(x + c)(x + d)$	M1	$ab = \pm 40$ or $2b + a = \pm 2$ $cd = \pm 20$ or $c + d = \pm 1$
	$2(x + 4)(x - 5)$	A1	$(2x + 8)(x - 5)$ and $(x + 4)(2x - 10)$ and $(x + 4)(x - 5)$ all score SC2 $(x - 4)(x + 5)$ scores SC1
4(b)	$(x + y)[(x + y) + (2x + 5y)]$	M1	
	$(x + y)(3x + 6y)$	A1	
	$3(x + y)(x + 2y)$	A1	$(x + y)(x + 2y)$ scores SC2
Alt 4(b)	$x^2 + xy + xy + y^2 + 2x^2 + 2xy + 5xy + 5y^2$ or $3x^2 + 9xy + 6y^2$	M1	Condone two errors
	$(x + y)(3x + 6y)$ or $(3x + 3y)(x + 2y)$ or $3(x^2 + 3xy + 2y^2)$	A1	
	$3(x + y)(x + 2y)$	A1	$(x + y)(x + 2y)$ scores SC2
5	$8c^3d^{12}$	B2	B1 For two out of three components correct

Q	Answer	Mark	Comments
6	$2y - 3x = 4$ $3y + 2x = -7$	M1	oe Rearrange into suitable form for elimination Allow one error
	$6y - 9x = 12$ and $6y + 4x = -14$ or $4y - 6x = 8$ and $9y + 6x = -21$	M1	oe Attempting to equate x or y coefficients Allow one error
	$13x = -26$ or $13y = -13$	M1	oe Correct elimination from their equations only award if ≤ 1 error on first two M marks
	$x = -2$ and $y = -1$	A1	
Alt 1 6	$2y = 3x + 4$ and $3y = -2x - 7$ or $3x = 2y - 4$ and $2x = -3y - 7$	M1	oe Rearrange into suitable form for elimination Allow one error
	$6y = 9x + 12$ and $6y = -4x - 14$ or $6x = 4y - 8$ and $6x = -9y - 21$	M1	oe Attempting to equate x or y coefficients Allow one error
	$0 = 13x + 26$ or $0 = 13y + 13$	M1	oe Correct elimination from their equations Only award if ≤1 error on first two M marks
	$x = -2$ and $y = -1$	A1	
Alt 2 6	$x = -1.5y - 3.5$	M1	oe Rearrange into suitable form for substitution Allow one error
	$2y = 3(-1.5y - 3.5) + 4$	M1	oe Substitution Allow one error
	$6.5y = -6.5$	M1	oe Correct simplification from their equation Only award if ≤1 error on first two M marks
	$y = -1$ and $x = -2$	A1	

Q	Answer	Mark	Comments
Alt 3 6	$y = 1.5x + 2$	M1	oe Rearrange into suitable form for substitution Allow one error
	$2x = -3(1.5x + 2) - 7$	M1	oe Substitution Allow one error
	$6.5x = -13$	M1	oe Correct simplification from their equation Only award if ≤1 error on first two M marks
	$x = -2$ and $y = -1$	A1	
7	Angle *CAD* = 46 or Angle *ACD* = 37 or Angle *CDE* = 83 or (37 + 46) or Angle *ADC* = 97 or 180 – (37 + 46)	M1	Any of these angles correctly marked or named ... could be on diagram
	Angle *DCE* = 46 or Angle *ACE* = 83 or (37 + 46)	M1	
	51	A1	
8(a)	$\frac{dy}{dx} = 3x^2 + 10x$	M1	Allow one error
	3 – 10 (= –7)	A1	$3 \times 1 + 10 \times -1$ is sufficient
8(b)	$(y =)\ (-1)^3 + 5(-1)^2 + 1$	M1	
	$(y =)$ 5	A1	
	Use of 'm' = –7 seen or implied	M1	Must be used in an equation
	y – their 5 = –7 $(x + 1)$	A1 ft	oe eg. $y = -7x - 2$

Q	Answer	Mark	Comments
9	1 : 2 : 5	B3	B2 For any ratio that is one step away from the answer eg $\sqrt{12} : 2\sqrt{12} : 5\sqrt{12}$ $\sqrt{1} : \sqrt{4} : \sqrt{25}$ 2 : 4 : 10 B1 For at least two of the three terms in their simplest form ie two of $2\sqrt{3} : 4\sqrt{3} : 10\sqrt{3}$ B1 For any correct equivalent ratio eg $\sqrt{2} : \sqrt{8} : \sqrt{50}$ $\sqrt{3} : \sqrt{12} : \sqrt{75}$

Q	Answer	Mark	Comments
10	$(5n - 3)^2 + 1$	M1	
	$25n^2 - 15n - 15n + 9 + 1$	M1	Allow 1 error Must have an n^2 term
	$25n^2 - 30n + 10$	A1	
	$5(5n^2 - 6n + 2)$	B1 ft	oe eg, shows that all terms divide by 5 or explains why the expression is a multiple of 5
Alt 1 10	Use of $an^2 + bn + c$ for terms of quadratic sequence ie, any one of $a + b + c = 5$ $4a + 2b + c = 50$ $9a + 3b + c = 145$	M1	
	$3a + b = 45$ $5a + b = 95$	M1	For eliminating c
	$25n^2 - 30n + 10$	A1	
	$5(5n^2 - 6n + 2)$	B1 ft	oe eg, shows that all terms divide by 5 or explains why the expression is a multiple of 5
Alt 2 10	5 50 145 290 45 95 145 2nd difference of 50 ÷ 2 (= 25)	M1	$25n^2$
	Subtracts their $25n^2$ from terms of sequence –20 –50 –80	M1	$-30n$
	$25n^2 - 30n + 10$	A1	
	$5(5n^2 - 6n + 2)$	B1ft	oe eg, shows that all terms divide by 5 or explains why the expression is a multiple of 5

Q	Answer	Mark	Comments
11(a)	Gradient $AC = \frac{4-0}{0-12}$ or $-\frac{1}{3}$	M1	oe
	$y = -\frac{1}{3}x + 4$	A1	oe eg $x + 3y = 4$ Must be an equation
11(b)	Gradient $OB = 3$	B1 ft	ft Their gradient in (a) using $m_1 \times m_2 = -1$
	Equation of OB: $y = 3x$	M1	ft Their gradient OB
	$3x = -\frac{1}{3}x + 4$	M1	ft Their equations
	$x = \frac{6}{5}$ or 1.2	A1 ft	oe (x coordinate of midpoint of OB) ft From their linear equations
	$y = \frac{18}{5}$ or 3.6	A1	oe (y coordinate of midpoint of OB)
	$\left(\frac{12}{5}, \frac{36}{5}\right)$ or (2.4, 7.2)	B1 ft	oe ft Their x and y values for the midpoint

Q	Answer	Mark	Comments
12(a)	Line $y = \frac{1}{2}x$ drawn	B1	Between $x = 0$ and $x = 4$
12(b)	Line $y = 2$ drawn	B1	Between $x = 0$ and $x = 4$
12(c)	$\left(\frac{dy}{dx} =\right)$ $6x^2 + a$	M1	Allow one error
	$x = -1$ $6 + a$	A1	
	$x = 2$ $24 + a$	A1	
	Their $(24 + a) = 2 \times$ their $(6 + a)$	M1	Must follow from their $\frac{dy}{dx}$ and must be an equation in a
	$a = 12$	A1	$a = -3$ from $\frac{dy}{dx} = 6x^2 + ax$ scores SC3

Q	Answer	Mark	Comments
13	$(x + 6)(x - 2)$	B1	
	$(x + 5)(x - 5)$	B1	
	$x(x - 5)$	B1	
	$\dfrac{\text{their } (x+6)(x-2)}{\text{their } (x+5)(x-5)} \times \dfrac{\text{their } x(x-5)}{x+6}$	M1	Must have attempted to factorise at least two of the above
	$\dfrac{x(x-2)}{x+5}$ or $\dfrac{x^2 - 2x}{x+5}$	A1	A0 if incorrect further work seen
14	$x = 8^{\frac{2}{3}}$ or $x = \sqrt[3]{64}$ or $x^3 = 64$ or $\sqrt{x} = 2$ or $x = 2^2$	M1	
	$x = 4$	A1	
	$y^2 = \dfrac{4}{25}$ or $\dfrac{1}{y^2} = \dfrac{25}{4}$ or $y^{-1} = \sqrt{\dfrac{25}{4}}$ or $\dfrac{1}{y} = \sqrt{\dfrac{25}{4}}$	M1	
	$y = \dfrac{2}{5}$ or $y^{-1} = \dfrac{5}{2}$ or $\dfrac{1}{y} = \dfrac{5}{2}$	A1	Accept $y = \pm\dfrac{2}{5}$ or $y^{-1} = \pm\dfrac{5}{2}$ or $\dfrac{1}{y} = \pm\dfrac{5}{2}$
	10	A1	

Q	Answer	Mark	Comments
15(a)	Correct use of Pythagoras' Theorem eg $YZ = \sqrt{2^2 - 1^2}$	M1	oe
	$X = 60°$ and $\sin X = \frac{\sqrt{3}}{2}$	A1	$X = 60°$ stated or 60° marked on diagram
15(b)	Correct use of Sine Rule $\frac{2-\sqrt{3}}{\sin A} = \frac{(4\sqrt{3}-6)}{\sin B}$	M1	oe
	$\sin B = \frac{(4\sqrt{3}-6)}{(2-\sqrt{3})} \times \frac{1}{4}$	M1	oe eg $\frac{(4\sqrt{3}-6)}{8-4\sqrt{3}}$ or $\frac{\sqrt{3}-1.5}{2-\sqrt{3}}$
	$= \frac{(4\sqrt{3}-6)(2+\sqrt{3})}{4(2-\sqrt{3})(2+\sqrt{3})}$	M1	For multiplying both numerator and denominator by conjugate of the form $a + \sqrt{b}$ … ft their expression for sin *B* eg $\frac{(4\sqrt{3}-6)(8+4\sqrt{3})}{(8-4\sqrt{3})(8+4\sqrt{3})}$ or $\frac{(\sqrt{3}-1.5)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$
	Numerator $= 8\sqrt{3} - 12 + 12 - 6\sqrt{3}$	A1 ft	eg $32\sqrt{3} - 48 + 48 - 24\sqrt{3}$ or $2\sqrt{3} - 3 + 3 - 1.5\sqrt{3}$
	Denominator $= 4$	A1 ft	eg 16 or 1
	$\sin B = \frac{\sqrt{3}}{2}$ and $B = 60°$	A1	Clearly shown
Alt 1 15 (b)	$\frac{CD}{4\sqrt{3}-6} = \frac{1}{4}$ or $CD = \frac{1}{4}(4\sqrt{3}-6)$	M1	oe where *D* is the foot of the perpendicular from *C* to *AB*
	$\sin B = \frac{\frac{1}{4}(4\sqrt{3}-6)}{2-\sqrt{3}}$	M1	
	$\frac{\frac{1}{4}(4\sqrt{3}-6)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$	M1	For multiplying both numerator and denominator by conjugate of the form $a + \sqrt{b}$ … ft their expression for sin *B*
	Numerator $= 2\sqrt{3} - 3 + 3 - 1.5\sqrt{3}$	A1 ft	
	Denominator $= 1$	A1 ft	
	$\sin B = \frac{\sqrt{3}}{2}$ and $B = 60°$	A1	Clearly shown

Q	Answer	Mark	Comments
Alt 2 15(b)	Correct use of Sine Rule $\frac{\sin A}{2-\sqrt{3}} = \frac{\sin B}{4\sqrt{3}-6}$	M1	oe
	$\frac{\sin A(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{\sin B}{4\sqrt{3}-6}$	M1	
	$\frac{\sin A(2+\sqrt{3})}{1} = \frac{\sin B}{4\sqrt{3}-6}$	A1	
	$\sin B = \frac{1}{4}(2+\sqrt{3})(4\sqrt{3}-6)$	M1	
	Numerator = $2\sqrt{3} - 3 + 3 - 1.5\sqrt{3}$	A1	
	$\sin B = \frac{\sqrt{3}}{2}$ and $B = 60°$	A1	Clearly shown

Q	Answer	Mark	Comments
16	$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ and $\frac{1}{\tan\theta} \equiv \frac{\cos\theta}{\sin\theta}$	M1	oe
	Denominator = $\sin\theta\cos\theta$	M1 Dep	oe
	$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ $(\sin^2\theta + \cos^2\theta \equiv 1)$ and $\frac{1}{\sin\theta\cos\theta}$	A1	All steps clearly shown